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Reports of the Department of Geodetic Science
Report No. 187

GEODETTIC SATELLITE OBSERVATIONS IN NORTH AMERICA (SOLUTION NA-9)

**CASE FILE
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by

Ivan I. Mueller, James P. Reilly, and Tomas Soler

Prepared for
National Aeronautics and Space Administration
Washington, D. C.

Contract No. NGR 36-008-093
OSURF Project No. 2514



The Ohio State University
Research Foundation
Columbus, Ohio 43212

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C., 20546.

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1. INTRODUCTION

The coordinates of several tracking stations tied to the NAD datum were computed at OSU through available observations to the GEOS-I satellite. Up to date results of the NA6 adjustment [Mueller et al., 1969] and NA8 adjustment [Mueller and Reilly, 1971] had been presented. The latter solution was performed using height constraints deduced from the SAO-69 geoid [Gaposchkin and Lambeck, 1970].

Recently a new detailed geoidal map with claimed accuracies of ± 2 m (on land), based on gravimetric and satellite data, was presented [Vincent, Strange and Marsh, 1971]. With the new geoid and the orthometric heights given in [NASA, 1971] more reliable height constraints were calculated and applied.

The basic purpose of this experiment was to compute the new solution NA9 by defining the origin of the system, from the point of view of error propagation, in the most favorable position applying "inner constraints" [Blaha, 1971], and imposing new weighted height constraints to all of the stations. The major differences with respect to formerly published adjustments can be seen in Table 1.

2. THEORETICAL BACKGROUND

2.1 Normal Equations for Optical Observations

The set of optical observations from MOTS and PC-1000 data had been previously screened [Mueller et al., 1969] and a set of reduced normal equations of the form

$$\dot{N}\dot{X} + \dot{U} = 0$$

obtained. The same symmetric coefficient matrix was used. This is composed of 3x3 blocks of the form [Mueller, 1968]

$$\dot{N}_{kk} = \sum_j M_{kj}^{-1} - \sum_j M_{kj}^{-1} (\sum_i M_{ij}^{-1})^{-1} M_{kj}^{-1} + P_k$$

$$\dot{N}_{k1} = -\sum_j M_{kj}^{-1} (\sum_i M_{ij}^{-1})^{-1} M_{i1}$$

where

$$M_{ij} = B_{ij} P_{ij}^{-1} B_{ij}^T$$

$$B_{ij} = S R_3 (-\alpha) R_2 (-90^\circ + \delta) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos \delta & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$S = \text{Solar motion matrix} = R_2(-x_p) R_1(-y_p) R_3 (\text{Sidereal Time})$$

and

k, l denotes particular ground stations

j denotes particular simultaneous event

i denotes any ground station participating in an event

\sum_i is the summation over all ground stations involved in event j

\sum_j is the summation over all events observed by ground station k and/or l

P_k
3x3 weight matrix associated with any particular ground station

P_{ij}
3x3 weight matrix of any observed direction

Table 1
General Information on the Geometric Adjustment

No. of PC-1000 stations	15
No. of MOTS stations	15
No. of SECOR stations	4
Total number of ground stations	34
σ { optical : $\sigma_b = \sigma_{\text{a cos } b} = 2.0$ range : $\sigma_r = 3.5$ m σ_o (a priori) = 1.0	Rejection criteria { optical : 6.3 range : none
Relative Position Constraints	ΔX ΔY ΔZ
3334 } Greenville, Mass.	-18.31 m 2.39 m 8.95 m
5333 }	
3648 } Hunter AFB, Ga.	-82.57 m -39.57 m -44.89 m
5649 }	
3861 } Homestead, Fla.	1700.69 m -564.69 m -1761.01 m
5861 }	

$$\sigma_{\Delta X} = \sigma_{\Delta Y} = \sigma_{\Delta Z} = 0.03 \text{ m}$$

	NA6	NA8	NA9
Station Constraint			
7073 Columbia, Mo.	$\sigma_x = \sigma_y = \sigma_z = 3.0 \text{ m}$		none
Chord Distance Constraint			
3861 Homestead, Fla. to 7043 Greenbelt, Md.	1,531.562.9 m $\sigma_d/D = 1/750,000$		1,531,560.39 m $\sigma_d/D = 1/750,000$
Height Constraint			
	none	Orthometric heights from [NASA, 1971]; Undulations from SAO69 geoid	Same as NA8, but undulations from [Vincent et al, 1971]. See σ_h in Table 4
Inner Constraints	none	none	yes
No. of degrees of freedom	5183	5254	5228
$\Sigma V'PV$	4962.6	5082	5039
$\hat{\sigma}_o$ (a posteriori)	.97	.98	.98

Finally, the vector of constant terms is expressed by

$$\dot{U}_k = -\sum_j M_{kj}^{-1} [X_k^o - (\sum_i M_{ij}^{-1})^{-1} \sum_i M_{ij}^{-1} X_i^o]$$

where as usual, the superscript (^o) denotes initial approximate values.

2.2 Normal Equations for Range Observations

The general form of the normal equations similarly to the optical case can be formulated as:

$$\tilde{N}\tilde{X} + \tilde{U} = 0$$

where the 3x3 blocks in \tilde{N} are now computed by [Mueller, 1968]

$$\tilde{N}_{kk} = \sum_j a_{kj}^T \tilde{p}_{kj} a_{kj} - \sum_j a_{kj}^T \tilde{p}_{kj} a_{kj} [\sum_i a_{ij}^T \tilde{p}_{ij} a_{ij}]^{-1} a_{kj}^T \tilde{p}_{kj} a_{kj}$$

$$\tilde{N}_{k1} = -\sum_j [a_{kj}^T \tilde{p}_{kj} a_{kj} (\sum_i a_{ij}^T \tilde{p}_{ij} a_{ij})^{-1} a_{ij}^T \tilde{p}_{ij} a_{ij}]$$

and the vector of constant terms having the form

$$\tilde{U}_k = -\sum_j a_{kj}^T \tilde{p}_{kj} \tilde{v}_{kj}$$

where

\tilde{v}_{kj} = Residual of any observed range from a particular station
(resulting from a preliminary least squares adjustment
of any simultaneous event with the stations held fixed)

p_{ij} = Weight of any observed range r_{ij}

$$a_{ij} = \left[\frac{X_j^o - X_i^o}{r_{ij}^o}, \frac{Y_j^o - Y_i^o}{r_{ij}^o}, \frac{Z_j^o - Z_i^o}{r_{ij}^o} \right]$$

and

X^o, Y^o, Z^o = Appropriate Cartesian coordinates in the average
terrestrial system.

Other subscripts and symbols have the same meaning as in the case of optical observations.

2.3 Constraint's Contributions to the Normal Equations

Two alternative definitions exist for the term "constraints". The absolute constraints represent certain conditions which have to be fulfilled exactly and with no uncertainties and the relative constraints (or weighted constraints) which have the same characteristics as the observations.

In general the contribution of the functional constraint equations

$$G(X, L_c) = 0$$

to the normal equations can be found bordering the normal equation matrix

$$\begin{bmatrix} N_{n-1} & C_n^T \\ C_n & -P_{c_n}^{-1} \end{bmatrix} \begin{bmatrix} X_n \\ -K_{c_n} \end{bmatrix} + \begin{bmatrix} U_{n-1} \\ W_n \end{bmatrix} = 0$$

from where after elimination of K_{c_n} it is easy to find

$$[N_{n-1} + C_n^T P_{c_n} C_n] X_n + U_{n-1} + C_n^T P_{c_n} W_n = 0$$

or

$$[N_{n-1} + N_n^c] X_n + U_{n-1} + U_n^c = 0 \quad (1)$$

where N_n^c and U_n^c are the contributions to the coefficient matrix and constant vector of the normal equation due to the application of constraints. The coefficient $n-1$ represents the normal equations of the previous set (without constraints).

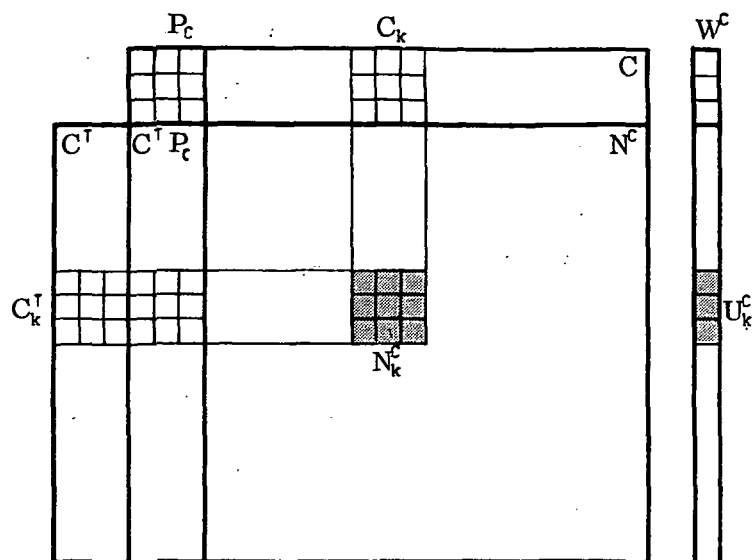
After the constraints are added the normal equations will take the usual form

$$N_n X_n + U_n = 0$$

and we are in the position to obtain the contribution from a new set of constraints.

Constraints can be applied between two stations k and l or to a single station. The contribution of these constraints to the matrix N (3×3 blocks) and U (3×1 blocks) can be schematically expressed in two different ways. Assuming that the matrix P_c is always diagonal it is possible to express:

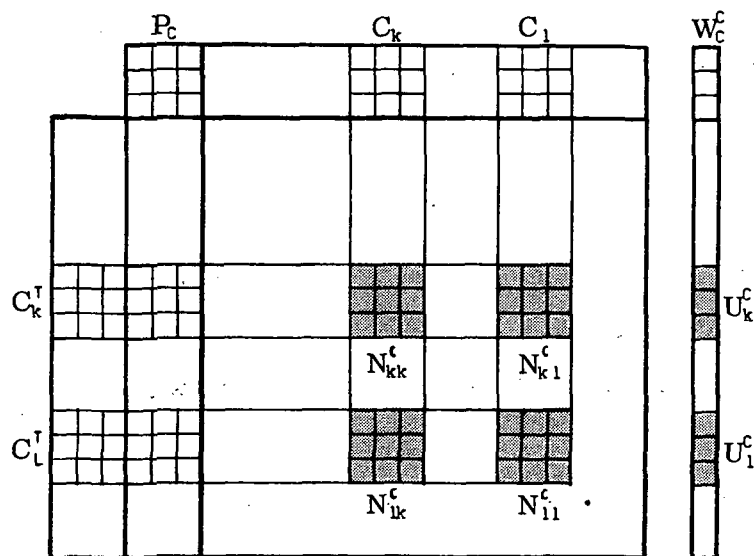
a) Contribution to the normals due to the constraint applied to station k.



$$N_k^c = C_k^T P_c C_k$$

$$U_k^c = C_k^T P_c W^c$$

b) Contribution to the normals due to the constraint between stations k and l



$$N_{kk}^c = C_k^T P_c C_k ; N_{kl}^c = C_k^T P_c C_l$$

$$U_k^c = C_k^T P_c W^c ; U_l^c = C_l^T P_c W^c$$

... (2)

These blocks obtained as indicated above for the corresponding case will be the only ones computed and added to the original normal equations as expressed by formula (1).

2.31 Relative Position Constraints

Relative position constraints were used in order to combine the normal equations from the optical and range mode. The relative constraints used

and their weights are given in Table 1. The expression for the combination of normals can be written as follows.

$$\begin{aligned} & [\bar{N} + N^{RC}] X + \bar{U} + U^{RC} = 0 \\ \text{or} \quad & \left\{ \begin{bmatrix} \dot{N} & 0 \\ 0 & \tilde{N} \end{bmatrix} + N^{RC} \right\} X + \begin{bmatrix} U \\ \tilde{U} \end{bmatrix} + U^{RC} = 0 \end{aligned}$$

where N^{RC} and U^{RC} computed from (2) are the contribution to the original normal equations ($\bar{N}X + \bar{U} = 0$).

In this case the functional constraint equations are

$$X_k - X_1 = \Delta X$$

$$Y_k - Y_1 = \Delta Y$$

$$Z_k - Z_1 = \Delta Z$$

Therefore:

$$\begin{matrix} C_k^{RC} = I & ; & C_1^{RC} = -I \\ 3 \times 3 & & 3 \times 3 \end{matrix}$$

$$\text{and} \quad \begin{matrix} U_k^{RC} = 0 & ; & U_1^{RC} = 0 \\ 3 \times 1 & & 3 \times 1 \end{matrix} \quad \text{because} \quad W^{RC} = G^{RC}(X^0, L_c^0) = 0$$

where

$$P_{RC} = \begin{bmatrix} \frac{1}{\sigma_{\Delta X}^2} & 0 & 0 \\ 0 & \frac{1}{\sigma_{\Delta Y}^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_{\Delta Z}^2} \end{bmatrix}$$

$$\text{and} \quad \begin{matrix} N_{kk}^{RC} = I P_{RC} I = P_{RC} \\ 3 \times 3 & & 3 \times 3 \end{matrix}$$

$$\begin{matrix} N_{11} = I P_{RC} I = P_{RC} \\ 3 \times 3 & & 3 \times 3 \end{matrix}$$

$$\begin{matrix} N_{k1}^{RC} = N_{1k}^{RC} = I P_{RC} (-I) = -P_{RC} \\ 3 \times 3 & & 3 \times 3 \end{matrix}$$

Thus, the diagonal elements of P_{RC} are added to each element of the diagonal of the blocks kk and ll of the matrix of the combined normals \bar{N} , and subtracted from the diagonal elements of the blocks kl and lk of \bar{N} . There is no contribution to the vector \bar{U} .

2.32 Length Constraints

The range observations introduced scale into our adjustment. In addition to the SECOR scale, a distance constraint was imposed between stations 3861 and 7043, whose updated Cape Canaveral datum coordinates were derived from the high precision geodimeter traverse in the eastern United States. Due to a recent correction in their coordinates a difference of 3 m from the previously used value (see Table 1) was taken into consideration [Meade, 1972].

The functional constraint equation is

$$G^{CH}(X, L_{CH}) = 0 \quad \text{or}$$

$$[(X_k - X_l)^2 + (Y_k - Y_l)^2 + (Z_k - Z_l)^2]^{\frac{1}{2}} = L_{kl}$$

$$C_{3 \times 1}^{CH} = \left[\frac{X_k^o - X_l^o}{L_{kl}^o}, \frac{Y_k^o - Y_l^o}{L_{kl}^o}, \frac{Z_k^o - Z_l^o}{L_{kl}^o} \right] \quad \text{and}$$

$$C_{3 \times 1}^{CH} = \left[-\frac{X_k^o - X_l^o}{L_{kl}^o}, -\frac{Y_k^o - Y_l^o}{L_{kl}^o}, -\frac{Z_k^o - Z_l^o}{L_{kl}^o} \right]$$

and

$$P_{CH}^1 = \frac{\sigma_{kl}^2}{\sigma_o^2} \quad \begin{array}{l} \sigma_{kl}^2 = \text{Variance of the chord} \\ \sigma_o^2 = \text{A priori variance of unit weight} \end{array}$$

Then the contribution to the normals are obtained applying (2)

$$N_{kk}^{CH} = (C_k^{CH})^T P_{CH} C_k^{CH}$$

3×3

$$N_{ll}^{CH} = (C_l^{CH})^T P_{CH} C_l^{CH}$$

3×3

$$N_{kl}^{CH} = (C_k^{CH})^T P_{CH} C_l^{CH}$$

3×3

$$U_k^{CH} = (C_k^{CH})^T P_{CH} W^{CH}$$

$$U_1^{CH} = (C_k^{CH})^T P_{CH} W^{CH}$$

The first three expressions in the above are added respectively to the blocks N_{kk} , N_{11} and N_{k1} of N ; the last two expressions are added respectively to the constant vectors U_k and U_1 .

2.33 Height Constraints

If the height of the station k is to be constraint, then

$$N_{kk}^H = (C_k^H)^T P_H C_k^H$$

where

$$C_k^H = [\cos \varphi_k^\circ \cos \lambda_k^\circ, \cos \varphi_k^\circ \sin \lambda_k^\circ, \sin \varphi_k^\circ]$$

and

$$P_H = \frac{1}{\sigma_{h_k}^2}$$

where φ_k° and λ_k° are the approximate geodetic coordinates and $\sigma_{h_k}^2$ is the variance of the height for station k (see Table 4).

The constant vector U^H can be computed from

$$U_k^H = (C_k^H)^T P_H W^H \quad \text{where}$$

$$W^H = h_k - h_k^\circ$$

At all stations a weighted height constraint was imposed in order to provide a valuable strengthening to the geometric network.

After the completion of the previous adjustments (NA6, NA8), a new geoid (see Figure 1) became available [Vincent, Strange and Marsh, 1971]. With the interpolated geoidal undulations for each station and their orthometric heights given in [NASA, 1971] reliable height constraints were computed by an iterative process self-explained in Figure 2 and summarized below.

$\Delta X = -41.1 \text{ m}$
 $\Delta Y = 189.3 \text{ m}$
 $\Delta Z = 158.0 \text{ m}$

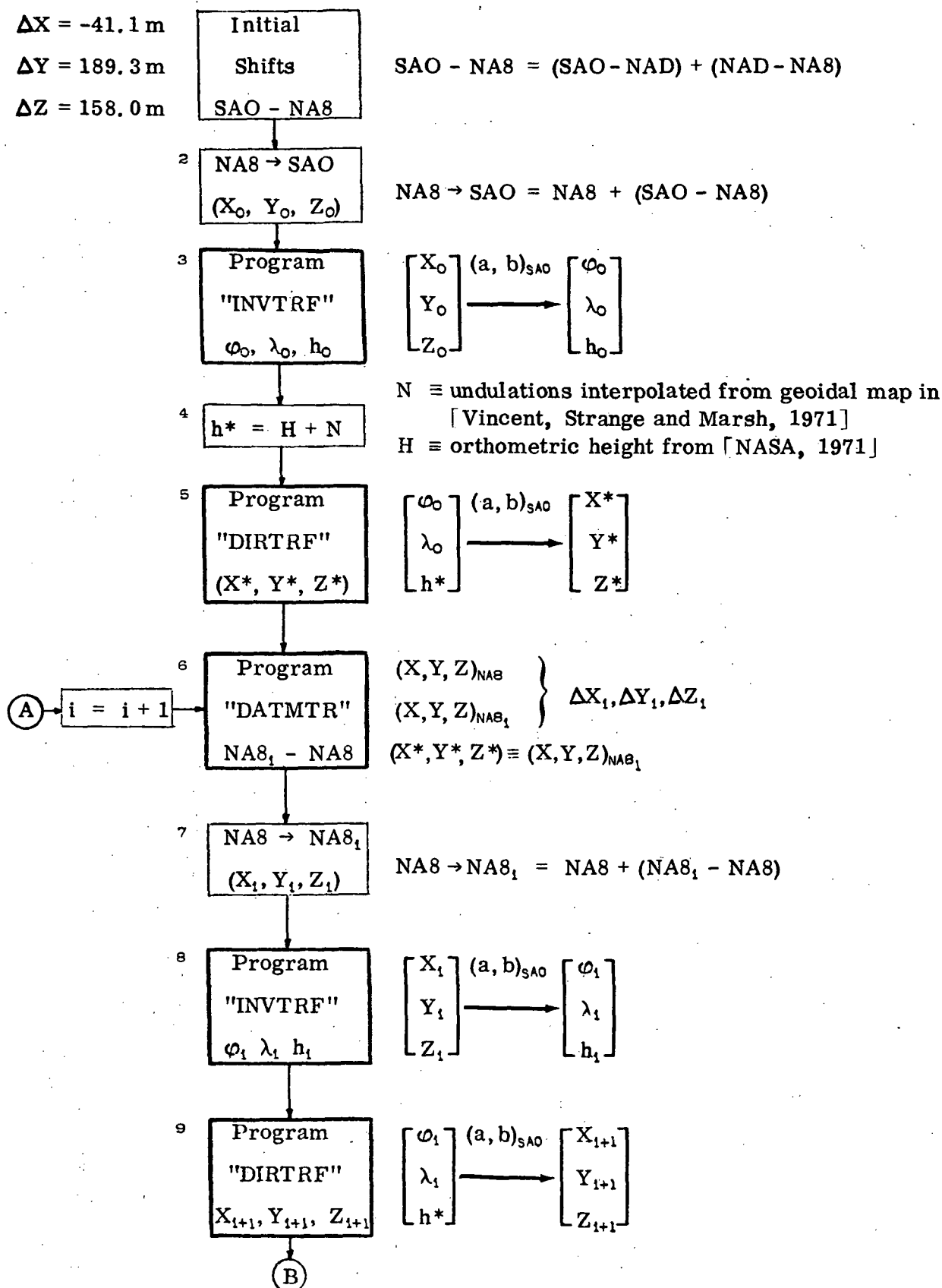


Figure 2

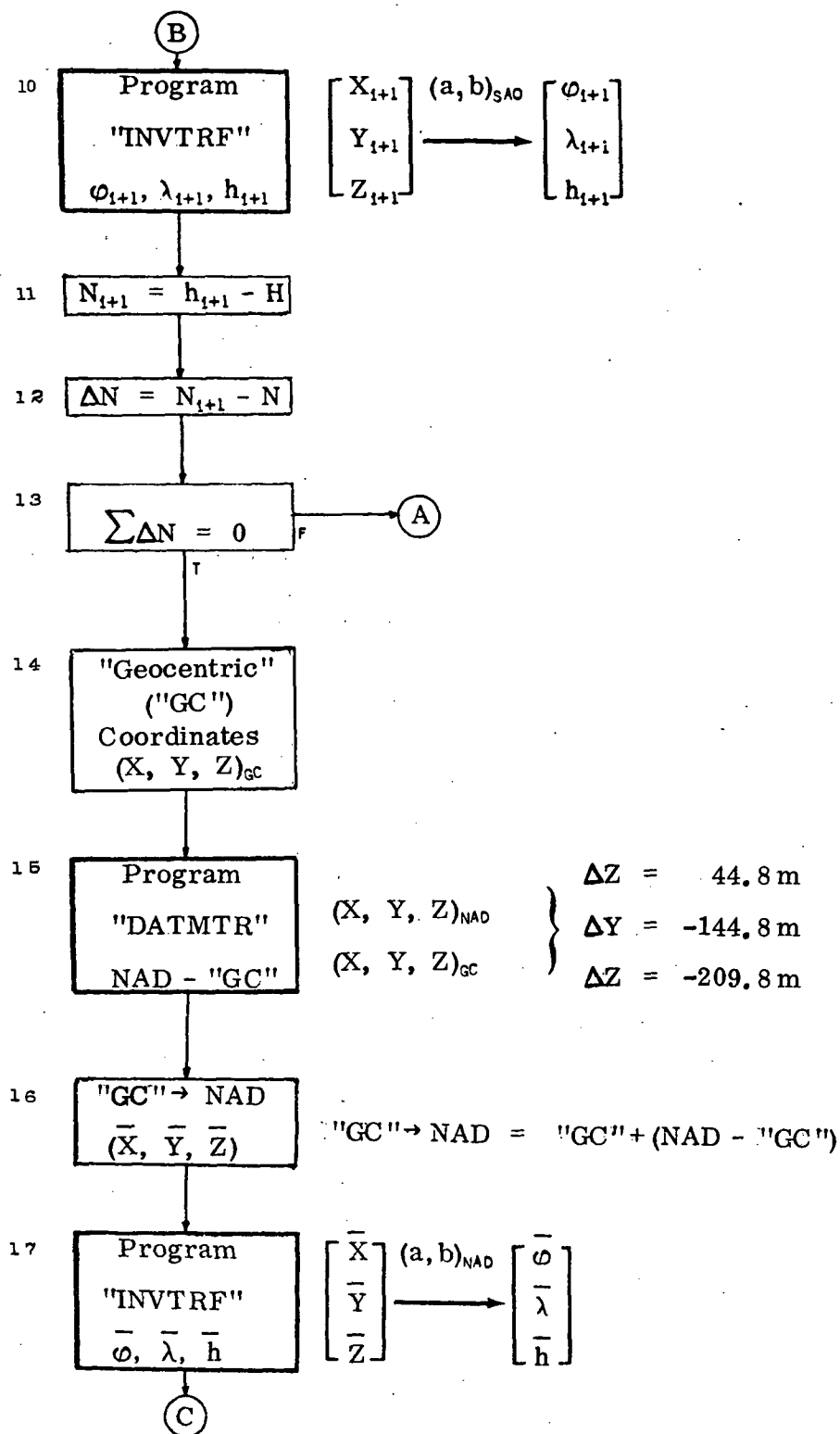


Figure 2 (continued)

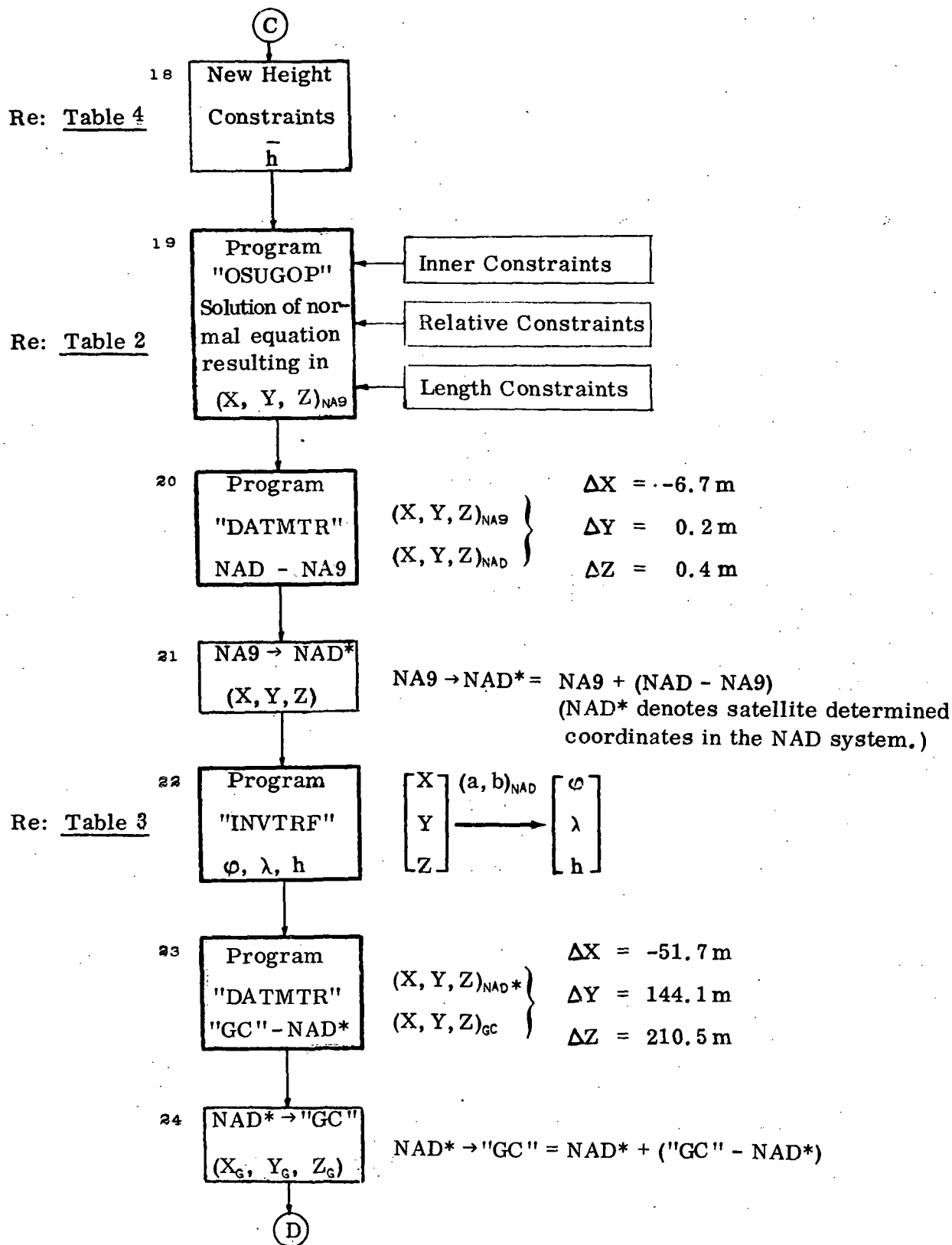


Figure 2 (continued)

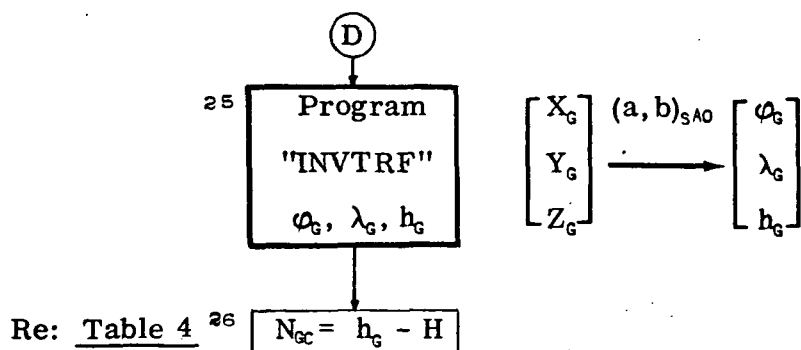


Figure 2 (continued)

From the initial values of the shifts SAO-NA8 (computed using the published shifts SAO-NAD and NAD-NA8 in [Mueller and Reilly, 1971]), the initial NA8 rectangular coordinates were shifted to the SAO-69 origin and geodetic coordinates (ϕ_o, λ_o, h_o) computed.

Ellipsoidal heights h^* then were calculated for each station using the undulations (N) from [Vincent, Strange and Marsh, 1971] (except for stations 1033 Fairbanks, Alaska and 3404 Swan Island whose undulations were interpolated from the SAO-69 geoid), and the sea level heights from [NASA, 1971]. With the original ϕ_o and λ_o , and with this new height h^* a new set of rectangular coordinates (with origin nearer to the geocenter) was obtained (X^*, Y^*, Z^*). From this set of coordinates and that of the NA8 new shift parameters ($\Delta X_1, \Delta Y_1, \Delta Z_1$) were computed and used to transform the NA8 set into NA8₁, which is nearer to the geocenter. Following this procedure iteratively, several shifts of this kind to the "geocenter" were performed until the sum of the undulation differences with respect to N approached zero. Through this process a "best" set of geocentric coordinates (GC) was obtained. Using these and the NAD coordinates of the stations, the shift parameters NAD-GC were calculated which when added to the GC coordinates resulted in Cartesian coordinates in the NAD system. From these the heights \bar{h} , to be constrained in the NA9 solution were computed (see step 18 in Figure 2 and Table 4).

2.34 Inner Constraints

Unlike in the NA8 adjustment, "inner constraints" (free adjustment technique) were also imposed in the NA9 solution in order to define the origin of the system in its most favorable position from the error propagation point of view [Blaha, 1971].

The functional inner constraints equations can be written as

$$C^I X = 0$$

where

$$C^I = \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 0 & | & \dots & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 0 & 1 & 0 & | & \dots & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 & | & \dots & | & 0 & 0 & 1 \end{bmatrix}$$

and C^I has as many 3×3 unit blocks as unknown points. X is the set of corrections of the approximate coordinates of the unknown points.

As it is known the "inner constraints" condition yields after the adjustment

$$\text{Trace } (\Sigma_x) = \text{minimum}$$

where Σ_x denotes the variance-covariance matrix of the coordinates of the ground stations following that the "inner adjustment" is the pseudo inverse solution by definition.

In the most general application when the "best" origin, orientation and scale are sought the matrix C^I has the form

$$C^I = \begin{bmatrix} C_1^I \\ C_2^I \\ C_3^I \end{bmatrix} = \begin{bmatrix} \begin{array}{ccc|ccc|ccc} & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & & & & & & & & & & & \\ & & & & & & & & & & & \\ \hline \end{array} \end{bmatrix}$$

where as usual ($^{\circ}$) expresses the approximate coordinates of the points.

If we represent the normal equations with the contribution of all the constraints (except inner constraints) by

$$[\bar{N} + N^{rc} + N^{ch} + N^H]X + \bar{U} + 0 + U^{ch} + U^H = 0 \quad \text{or}$$

$$NX + U = 0$$

then the inner adjustment can be obtained by bordering the coefficient matrix N of the normal equations as

$$\begin{bmatrix} N & (C^I)^T \\ C^I & 0 \end{bmatrix} \begin{bmatrix} X \\ -K_I \end{bmatrix} = \begin{bmatrix} -U \\ 0 \end{bmatrix}$$

Upon the addition of any kind of constraint to the normal equations, it becomes necessary to consider also its contribution to $\Sigma V^T P V$. The degrees of freedom change as well. In order to compute the proper variance of unit weight the latter must be taken into consideration.

3. THE SOLUTION

With the specific constraints mentioned above the NA9 solution was computed using the general OSUGOP program (from step 19 down in Figure 2) [Reilly et al., 1972].

The coordinates of the NA9 solution are presented in Table 2 with their corresponding standard deviations. The coordinates transformed to the NAD are shown in Table 3.

Table 4 presents the constrained heights (\bar{h}) used at each station, the final undulations (N_{gc}) and their differences (ΔN), compared with those published in [Vincent, Strange and Marsh, 1971]. (For the procedure see steps 23 to 26 in Figure 2). In column ΔN and in parenthesis, the differences published previously in [Vincent, Strange and Marsh, 1971] are also shown. It can be seen that only two stations show substantial disagreements (3903 Herndon, Virginia and 3407 Trinidad). It seems likely that the orthometric height as given in [NASA, 1971] for station 3903 has a gross error. The discrepancy with respect to station 3407 may be due to the fact that it is situated in the Caribbean, where large geoidal gradients being present, the interpolation is uncertain.

4. COMPARISON WITH OTHER SOLUTIONS

Table 5 summarizes the transformation parameters (systematic differences) between the NA9, the "GC", the NAD and the SAO systems (the latter from [Gaposchkin and Lambeck, 1970]). Two sets of parameters are listed. The first was obtained through the assumption that only translation exists between the sets of coordinates. The second set presents the general seven parameter solution. In this latter transformation, first the rotations were computed from direction cosines independent of the origins and scales. The scale factors were calculated from the chord distances independent of the origins and orientations. These

parameters (rotations and scale) constrained with their variances were used in the solution for all seven parameters [Kumar, 1972]. Tables 6 to 9 show the constrained solutions with the resulting variance-covariance matrices and the correlation coefficient matrices. In the variance-covariance matrices the angular units are in radians.

5. CONCLUSIONS

The solution for the station coordinates (Tables 2 and 3) appear to be completely valid. The standard deviations of the coordinates are all acceptable being more realistic than those of previous adjustments especially in the geodetic heights. The obtained origin for the "GC" system seems to be very near the geocenter, despite its geometric deviation.

Table 2

Coordinates of the North American GEOS-I Tracking Stations from the NA-9 Free Geometric Adjustment

Station	Name		NA-9	σ	Station	Name		NA-9	σ
1021	Blossom Point, Maryland MOTS 40	X Y Z	1, 118, 058.7 -4, 876, 470.5 3, 942, 797.5	3.5 3.1 3.1	3334	Greenville, Mississippi PC-1000	X Y Z	-84, 952.3 -5, 328, 109.6 3, 493, 270.1	4.8 4.1 5.6
1022	Ft. Myers, Florida SECOR	X Y Z	807, 891.3 -5, 652, 138.1 2, 833, 333.5	2.6 1.8 1.9	3400	Colorado Springs, Colorado PC-1000	X Y Z	-1, 275, 160.4 -4, 798, 170.6 3, 994, 031.3	9.8 6.0 5.9
1030	Mojave, California MOTS 40	X Y Z	-2, 357, 204.5 -4, 646, 475.3 3, 668, 126.2	6.6 3.6 2.9	3401	LG.HanscomField Massachusetts PC-1000	X Y Z	1, 513, 173.5 -4, 463, 721.1 4, 282, 882.5	4.0 4.8 3.6
1032	St. Johns, Newfoundland MOTS 40	X Y Z	2, 602, 613.4 -3, 419, 498.3 4, 697, 432.8	50.2 60.3 17.8	3402	Semmes, Alabama PC-1000	X Y Z	167, 298.1 -5, 482, 118.2 3, 244, 869.0	3.7 2.4 3.0
1033	College, Alaska MOTS 40	X Y Z	-2, 299, 223.8 -1, 445, 844.4 5, 751, 612.5	11.7 33.0 9.8	3404	Swan Island PC-1000	X Y Z	642, 529.1 -6, 054, 080.9 1, 895, 525.8	4.5 4.1 4.9
1034	E. Grand Fork, Minnesota MOTS 40	X Y Z	-521, 668.9 -4, 242, 203.4 4, 718, 539.2	3.3 4.1 3.7	3405	Grand Turk PC-1000	X Y Z	1, 919, 526.9 -5, 621, 239.1 2, 315, 604.3	5.3 3.7 4.9
1042	Rosman, N.C. MOTS 40	X Y Z	647, 535.9 -5, 178, 081.5 3, 656, 535.8	2.9 2.4 2.7	3406	Curacao PC-1000	X Y Z	2, 251, 840.5 -5, 817, 062.4 1, 327, 044.7	5.3 3.5 6.7
3106	Antigua Island PC-1000	X Y Z	2, 881, 873.1 -5, 372, 311.3 1, 868, 379.9	6.6 3.2 5.0	3407	Trinidad PC-1000	X Y Z	2, 979, 924.1 -5, 513, 690.3 1, 180, 997.4	8.2 4.6 8.9

All coordinates and standard deviations in meters.

Table 2 (continued)

Station	Name	NA-9	σ	Station	Name	NA-9	σ
3648	Hunter AFB, Georgia PC-1000	X Y Z 832, 601.2 -5, 349, 695.7 3, 360, 417.5	3.4 2.0 1.9	7036	Edinburg, Texas MOTS 40	X Y Z -828, 452.8 -5, 657, 613.0 2, 816, 643.5	3.8 2.3 2.8
3657	Aberdeen, Maryland PC-1000	X Y Z 1, 186, 824.6 -4, 785, 338.3 4, 032, 712.1	3.6 3.6 3.2	7037	Columbia, Missouri MOTS 40	X Y Z -191, 252.8 -4, 967, 435.7 3, 983, 080.9	2.9 2.6 2.8
3861	Homestead AFB, Florida PC-1000	X Y Z 961, 809.0 -5, 679, 305.3 2, 729, 717.7	3.2 1.8 2.1	7039	Bermuda Island MOTS 40	X Y Z 2, 308, 250.7 -4, 873, 750.2 3, 394, 386.6	5.7 3.9 3.7
3902	Cheyenne, Wyoming PC-1000	X Y Z -1, 234, 651.2 -4, 651, 377.8 4, 174, 580.2	9.5 7.5 6.6	7040	San Juan, P.R. MOTS 40	X Y Z 2, 465, 089.9 -5, 535, 083.6 1, 985, 352.6	5.7 3.3 4.2
3903	Herndon, Virginia PC-1000	X Y Z 1, 089, 019.6 -4, 843, 098.5 3, 991, 566.0	10.9 13.8 9.6	7043	GSFC, Greenbelt, Maryland PTH-100	X Y Z 1, 130, 747.2 -4, 831, 473.8 3, 993, 963.6	3.5 2.7 2.3
5001	Herndon, Virginia SECOR	X Y Z 1, 088, 888.8 -4, 843, 086.2 3, 991, 668.3	7.1 3.4 4.4	7045	Denver, Colorado MOTS 40	X Y Z -1, 240, 427.2 -4, 760, 380.6 4, 048, 803.0	4.6 3.4 3.3
5333	Stoneville Mississippi SECOR	X Y Z -84, 970.6 -5, 328, 107.3 3, 493, 279.1	4.8 4.1 5.6	7072	Jupiter, Florida MOTS 40	X Y Z 976, 302.8 -5, 601, 548.0 2, 880, 074.7	3.1 2.3 2.5
5649	Hunter AFB, Georgia SECOR	X Y Z 832, 518.5 -5, 349, 735.3 3, 360, 372.6	3.4 2.0 1.9	7075	Sudbury, Ontario MOTS 40	X Y Z 692, 653.0 -4, 347, 221.5 4, 600, 299.3	4.2 4.7 4.2
5861	Homestead AFB, Florida SECOR	X Y Z 963, 509.7 -5, 679, 870.0 2, 727, 956.7	3.2 1.8 2.1	7076	Jamaica, B.W.I. MOTS 40	X Y Z 1, 384, 194.3 -5, 905, 810.3 1, 966, 381.5	4.8 3.1 5.3

General Information:

No. of ground stations 34
 No. of height constraints 34
 No. of chord constraints 1

No. of degrees of freedom

5228

Quadratic sum of the residuals ($V^T PV$)

5039

Standard deviation of unit weight

0.98

Table 3

NAD Coordinates of the North American GEOS-I Tracking Stations

Station	Name	ϕ	λ	NAD	σ	Station	Name	ϕ	λ	NAD	σ
1021	Blossom Point, Maryland MOTS 40	ϕ	λ	38° 25' 49.81 282 54 47.80 6.4 m	0.11 0.15 2.3 m	3334	Greenville, Mississippi PC-1000	ϕ	λ	33° 25' 31.40 269 5 11.24	0.21 0.18
1022	Ft. Myers, Florida SECOR	ϕ	λ	26 32 52.07 278 8 3.91 24.5	0.06 0.09 1.7	3400	Colorado Springs, Colorado PC-1000	ϕ	λ	39 0 22.22 255 7 1.29 2190.0	0.24 0.40 4.6
1030	Mojave, California MOTS 40	ϕ	λ	35 19 47.92 243 6 2.81 900.8	0.10 0.28 2.4	3401	L.G.Hanscom Field Massachusetts PC-1000	ϕ	λ	42 27 18.01 288 43 34.46 82.2	0.14 0.20 3.4
1032	St. Johns, Newfoundland MOTS 40	ϕ	λ	47 44 27.90 307 16 30.26 99.0	0.83 3.65 4.9	3402	Semmes, Alabama PC-1000	ϕ	λ	30 46 49.58 271 44 52.33 79.8	0.10 0.14 2.3
1033	College, Alaska MOTS 40	ϕ	λ	64 52 19.60 212 9 47.92 148.6	0.47 2.43 9.5	3404	Swan Island PC-1000	ϕ	λ	17 24 17.13 276 3 29.28 56.1	0.16 0.15 4.1
1034	E. Grand Fork, Minnesota MOTS 40	ϕ	λ	48 1 21.19 262 59 21.71 255.2	0.16 0.17 2.2	3405	Grand Turk PC-1000	ϕ	λ	21 25 46.80 288 51 13.68 -5.4	0.16 0.19 3.6
1042	Rosman, N.C. MOTS 40	ϕ	λ	35 12 7.03 277 7 40.55 914.2	0.09 0.12 2.1	3406	Curacao PC-1000	ϕ	λ	12 5 23.36 291 9 42.53 38.8	0.22 0.18 3.4
3106	Antigua Island PC-1000	ϕ	λ	17 8 52.81 298 12 37.29 5.4	0.17 0.23 2.4	3407	Trinidad PC-1000	ϕ	λ	10 44 32.60 298 23 21.59 268.5	0.29 0.28 3.9

Note: The above coordinates were arrived at by applying the shifts $\Delta X = -6.7\text{m}$, $\Delta Y = 0.2\text{m}$ and $\Delta Z = 0.4\text{m}$ to the NA-9 coordinates and then converting these values to ellipsoidal coordinates on the ellipsoid $a = 6378206.4\text{m}$, $b = 6356583.8\text{m}$.

Table 3(continued)

Station	Name		NAD	σ
3648	Hunter AFB, Georgia PC-1000	ϕ λ h	32° 0' 5".88 278 50 46.26 23.7m	0.08 0.13 1.4m
3657	Aberdeen, Maryland PC-1000	ϕ λ h	39 28 19.25 283 55 44.15 6.2	0.12 0.16 2.5
3861	Homestead AFB, Florida PC-1000	ϕ λ h	25 30 25.08 279 36 43.00 15.2	0.07 0.12 1.7
3902	Cheyenne, Wyoming PC-1000	ϕ λ h	41 7 58.01 255 8 3.32 1883.0	0.29 0.40 4.9
3903	Herndon, Virginia PC-1000	ϕ λ h	38 59 34.36 282 40 21.58 95.1	0.34 0.45 13.4
5001	Herndon, Virginia SECOR	ϕ λ h	38 59 37.78 282 40 16.40 127.7	0.16 0.29 2.7
5333	Stoneville Mississippi SECOR	ϕ λ h	33 25 31.68 269 5 10.53 46.4	0.21 0.18 2.0
5649	Hunter AFB, Georgia SECOR	ϕ λ h	32 0 4.19 278 50 42.92 22.3	0.08 0.13 1.4
5861	Homestead AFB, Florida SECOR	ϕ λ h	25 29 21.66 279 37 39.66 16.2	0.07 0.12 1.7

Station	Name		NAD	σ
7036	Edinburg, Texas MOTS 40	ϕ λ h	26° 22' 45".46 261 40 9.19 75.2	0.09 0.13 2.3
7037	Columbia, Missouri MOTS 40	ϕ λ h	38 53 35.84 267 47 42.13 276.5	0.10 0.12 2.1
7039	Bermuda Island MOTS 40	ϕ λ h	32 21 48.92 295 20 33.20 23.1	0.13 0.24 2.6
7040	San Juan, P.R. MOTS 40	ϕ λ h	18 15 26.46 294 0 21.85 58.4	0.14 0.20 3.0
7043	GSFC, Greenbelt, Maryland PTH-100	ϕ λ h	39 1 15.58 283 10 19.91 50.9	0.08 0.15 2.4
7045	Denver, Colorado MOTS 40	ϕ λ h	39 38 48.06 255 23 41.80 1793.0	0.12 0.20 2.4
7072	Jupiter, Florida MOTS 40	ϕ λ h	27 1 13.29 279 53 12.39 26.3	0.08 0.12 2.1
7075	Sudbury, Ontario MOTS 40	ϕ λ h	46 27 21.11 279 3 10.33 278.0	0.18 0.21 2.6
7076	Jamaica, B.W.I. MOTS 40	ϕ λ h	18 4 32.60 283 11 26.57 474.8	0.18 0.16 2.7

Table 4

Height Constraints and Undulations
(all units in meters)

Number	Station	Constraints		N		ΔN
		h	σ_h	N_{gc}^*	[Vincent et al., 1971]	
1021	Blossom Pt., Md.	9	3	- 27	-26	- 1 (- 7)
1022	Fort Myers, Florida	23	3	- 16	-18	2 (1)
1030	Goldstone, Calif.	898	3	- 23	-27	4 (8)
1032	St. John's, Nswf.	102	5	12	13	- 1
1033*	Fairbanks, Alaska	145	10	16		
1034	E. Grand Forks, Minn.	251	3	- 13	-18	5 (11)
1042	Rosman, N.C.	916	3	- 23	-22	- 1 (- 3)
3106	Antigua, W.I.	12	3	- 45	-40	- 5 (- 2)
3334	Stoneville, Mississippi	45	3	- 20	-19	- 1
3400	Colorado Springs, Col.	2184	5	- 4	-10	6
3401	Bedford, Mass.	89	5	- 27	-21	- 6
3402	Semmes, Alabama	84	3	- 21	-18	- 3 (-12)
3404*	Swan Island	79	7	- 32		
3405	Grand Turk, B.I.	0	5	- 51	-47	- 4 (-27)
3406	Curacao, N. Antilles	44	5	- 30	-26	- 4
3407	Trinidad, Tobago	285	5	- 50	-34	-16
3648	Hunter AFB, Georgia	19	3	- 19	-24	5 (- 5)
3657	Aberdeen, Maryland	7	3	- 27	-26	- 1 (- 4)
3861	Homestead, Florida	16	3	- 22	-22	0
3902	Cheyenne, Wyoming	1882	5	- 8	-10	2
3903	Herndon, Virginia	132	3	-100	-26	-74
5001	Herndon, Virginia	132	3	- 27	-26	- 1
5333	Stoneville, Mississippi	45	3	- 17	-19	2
5649	Hunter AFB, Georgia	23	5	- 23	-23	0
5861	Homestead, Florida	22	3	- 27	-22	- 5 (- 5)
7036	Edinburg, Texas	72	3	- 8	-11	3
7037	Columbia, Missouri	270	3	- 17	-24	7 (10)
7039	Bermuda	26	3	- 38	-36	- 2 (- 2)
7040	San Juan, P.R.	57	5	- 40	-41	1 (5)
7043	Greenbelt, Maryland	56	3	- 30	-26	- 4 (-15)
7045	Denver, Colorado	1787	3	- 7	-10	3 (16)
7072	Jupiter, Florida	26	3	- 23	-24	1 (1)
7075	Sudburg, Canada	276	3	- 28	-31	3 (20)
7076	Kingston, Jamaica	473	3	- 20	-23	3 (20)

*The "geocentric" coordinates were obtained from the NA-9 by adding the following shifts: $\Delta X = -51.7$ m, $\Delta Y = 144.1$ m, $\Delta Z = 210.5$ m.

* Undulations from SAO 69 geoid.

Table 5

Transformation Parameters

		NA9-NAD	NA9-"GC"	NA9-SAO	"GC"-NAD
No Stations		28	34	11	32
3 param. transf.	$\Delta X(m)$	6.7 ± 1.3	51.7 ± 0.7	35.3 ± 2.6	-44.8 ± 1.3
	$\Delta Y(m)$	-0.2 ± 1.1	-144.1 ± 0.8	-148.3 ± 2.6	144.8 ± 1.1
	$\Delta Z(m)$	-0.4 ± 1.2	-210.5 ± 0.9	-175.0 ± 2.6	209.8 ± 1.2
7 parameter transf.*	$\Delta X(m)$	-3.5 ± 2.4	40.0 ± 1.6	25.9 ± 6.6	-38.0 ± 2.2
	$\Delta Y(m)$	-18.4 ± 2.5	-142.9 ± 1.1	-200.7 ± 7.9	125.5 ± 2.3
	$\Delta Z(m)$	19.1 ± 2.5	-204.1 ± 1.5	-172.5 ± 7.6	226.7 ± 2.3
	$\theta_z(^{\circ})$	-0.83 ± 0.06	-0.34 ± 0.04	-0.62 ± 0.19	-0.36 ± 0.06
	$\theta_y(^{\circ})$	0.53 ± 0.05	-0.20 ± 0.03	0.14 ± 0.16	0.83 ± 0.04
	$\theta_x(^{\circ})$	-0.14 ± 0.08	-0.21 ± 0.05	0.94 ± 0.24	-0.10 ± 0.07
	$\epsilon(\times 10^6)$	-4.23 ± 0.32	-0.15 ± 0.05	-6.90 ± 1.20	-4.04 ± 0.29

*Rotation and scale factor parameters constrained.

Table 6

TRANSFORMATION PARAMETERS: NA9-NAD

Scale Factor and Rotation Parameters Constrained to Independently Determined Values

ΔX	ΔY	ΔZ	ϵ	θ_z	θ_y	θ_x
METERS	METERS	METERS	(10.0+5)	SECONDS	SECONDS	SECONDS
-3.51	-18.36	19.09	-4.23	-0.83	0.53	-0.14

VARIANCE - COVARIANCE MATRIX

0.573D+01	0.123D+01	0.192D+01	-0.374D-07	0.526D-06	0.272D-06	-0.365D-06
0.123D+01	0.606D+01	0.724D+00	0.512D-06	0.222D-06	0.758D-07	-0.506D-06
0.192D+01	0.724D+00	0.606D+01	-0.364D-06	0.290D-06	0.924D-07	-0.735D-06
-0.374D-07	0.512D-06	-0.364D-06	0.100D-12	-0.839D-15	0.383D-17	0.103D-14
0.526D-06	0.222D-06	0.290D-06	-0.839D-15	0.922D-13	0.160D-13	-0.573D-13
0.272D-06	0.758D-07	0.924D-07	0.383D-17	0.160D-13	0.541D-13	-0.207D-13
-0.365D-06	-0.506D-06	-0.735D-06	0.103D-14	-0.573D-13	-0.207D-13	0.145D-12

COEFFICIENTS OF CORRELATION

0.100D+01	0.208D+00	0.326D+00	-0.493D-01	0.723D+00	0.489D+00	-0.400D+00
0.208D+00	0.100D+01	0.119D+00	0.657D+00	0.297D+00	0.132D+00	-0.540D+00
0.326D+00	0.119D+00	0.100D+01	-0.467D+00	0.388D+00	0.161D+00	-0.785D+00
-0.493D-01	0.657D+00	-0.467D+00	0.100D+01	-0.872D-02	0.521D-04	0.851D-02
0.723D+00	0.297D+00	0.388D+00	-0.872D-02	0.100D+01	0.227D+00	-0.495D+00
0.489D+00	0.132D+00	0.161D+00	0.521D-04	0.227D+00	0.100D+01	-0.234D+00
-0.400D+00	-0.540D+00	-0.785D+00	0.851D-02	-0.495D+00	-0.234D+00	0.100D+01

Table 7

TRANSFORMATION PARAMETERS: NA9-GC

Scale Factor and Rotation Parameters Constrained to Independently Determined Values

ΔX	ΔY	ΔZ	ϵ	θ_z	θ_y	θ_x
METERS	METERS	METERS	(10.0+5)	SECONDS	SECONDS	SECONDS
40.00	-142.92	-204.13	-0.15	-0.34	-0.20	-0.21

VARIANCE - COVARIANCE MATRIX

0.2680+01	0.6210+00	0.6860+00	-0.1720-08	0.2240-06	0.1420-06	-0.1470-06
0.6210+00	0.1290+01	0.9360+00	0.1380-07	0.9630-07	0.3840-07	-0.1920-06
0.6860+00	0.9360+00	0.2120+01	-0.8570-08	0.1080-06	0.3470-07	-0.2870-06
-0.1720-08	0.1380-07	-0.8570-08	0.2580-14	-0.4700-17	0.6590-17	0.1290-17
0.2240-06	0.9630-07	0.1080-06	-0.4700-17	0.3760-13	0.8300-14	-0.2160-13
0.1420-06	0.3840-07	0.3470-07	0.6590-17	0.8300-14	0.2950-13	-0.1020-13
-0.1470-06	-0.1920-06	-0.2870-06	0.1290-17	-0.2160-13	-0.1020-13	0.5600-13

COEFFICIENTS OF CORRELATION

0.1000+01	0.3350+00	0.2880+00	-0.2080-01	0.7060+00	0.5050+00	-0.3790+00
0.3350+00	0.1000+01	0.5670+00	0.2390+00	0.4380+00	0.1970+00	-0.7150+00
0.2880+00	0.5670+00	0.1000+01	-0.1160+00	0.3830+00	0.1390+00	-0.8340+00
-0.2080-01	0.2390+00	-0.1160+00	0.1000+01	-0.4770-03	0.7560-03	0.1070-03
0.7060+00	0.4380+00	0.3830+00	-0.4770-03	0.1000+01	0.2490+00	-0.4710+00
0.5050+00	0.1970+00	0.1390+00	0.7560-03	0.2490+00	0.1000+01	-0.2510+00
-0.3790+00	-0.7150+00	-0.8340+00	0.1070-03	-0.4710+00	-0.2510+00	0.1000+01

Table 8

TRANSFORMATION PARAMETERS: NA9-SAO

Scale Factor and Rotation Parameters Constrained to Independently Determined Values

ΔX	ΔY	ΔZ	ϵ	θ_z	θ_y	θ_x
METERS	METERS	METERS	(10.0+5)	SECONDS	SECONDS	SECONDS
25.89	-200.72	-172.54	-6.90	-0.62	0.14	0.94

VARIANCE - COVARIANCE MATRIX

0.4410+02	0.8000+01	0.1290+02	-0.6440-06	0.5050-05	0.3180-05	-0.2420-05
0.8000+01	0.6220+02	-0.2310+01	0.7300-05	0.1810-05	0.6180-06	-0.4980-05
0.1290+02	-0.2310+01	0.5820+02	-0.5330-05	0.1800-05	0.4220-06	-0.6450-05
-0.6440-06	0.7300-05	-0.5330-05	0.1450-11	0.2200-14	0.1440-14	-0.4320-14
0.5050-05	0.1810-05	0.1800-05	0.2200-14	0.6760-12	0.1860-12	-0.3800-12
0.3180-05	0.6180-06	0.4220-06	0.1440-14	0.1860-12	0.6100-12	-0.1430-12
-0.2420-05	-0.4980-05	-0.6450-05	-0.4320-14	-0.3800-12	-0.1430-12	0.1310-11

COEFFICIENTS OF CORRELATION

0.1000+01	0.1530+00	0.2550+00	-0.8040-01	0.8110+00	0.6120+00	-0.3180+00
0.1530+00	0.1000+01	-0.3830-01	0.7680+00	0.2460+00	0.1000+00	-0.5520+00
0.2550+00	-0.3830-01	0.1000+01	-0.5790+00	0.2520+00	0.7080-01	-0.7390+00
-0.8040-01	0.7680+00	-0.5790+00	0.1000+01	0.1950-02	0.1530-02	-0.3130-02
0.8110+00	0.2460+00	0.2520+00	0.1950-02	0.1000+01	0.2540+00	-0.3540+00
0.6120+00	0.1000+00	0.7080-01	0.1530-02	0.2540+00	0.1000+01	-0.1600+00
-0.3180+00	-0.5520+00	-0.7390+00	-0.3130-02	-0.3540+00	-0.1600+00	0.1000+01

Table 9

TRANSFORMATION PARAMETERS: "GC"NAD

Scale Factor and Rotation Parameters Constrained to Independently Determined Values

ΔX	ΔY	ΔZ	ϵ	θ_z	θ_y	θ_x
METERS	METERS	METERS	(10.0+5)	SECONDS	SECONDS	SECONDS
-38.03	125.48	226.72	-4.04	-0.36	0.83	-0.10

VARIANCE - COVARIANCE MATRIX

0.4830+01	0.9720+00	0.1530+01	-0.3430-07	0.4300-06	0.2300-06	-0.2880-06
0.9720+00	0.5100+01	0.4860+00	0.4460-06	0.1790-06	0.6160-07	-0.4100-06
0.1530+01	0.4860+00	0.5070+01	-0.3120-06	0.2270-06	0.7180-07	-0.5930-06
-0.3430-07	0.4460-06	-0.3120-06	0.8670-13	-0.5780-15	0.2210-16	0.6810-15
0.4300-06	0.1790-06	0.2270-06	-0.5780-15	0.7500-13	0.1310-13	-0.4490-13
0.2300-06	0.6160-07	0.7180-07	0.2210-16	0.1310-13	0.4640-13	-0.1660-13
-0.2880-06	-0.4100-06	-0.5930-06	0.6810-15	-0.4490-13	-0.1660-13	0.1170-12

COEFFICIENTS OF CORRELATION

0.1000+01	0.1960+00	0.3090+00	-0.5290-01	0.7140+00	0.4850+00	-0.3840+00
0.1960+00	0.1000+01	0.9550-01	0.6710+00	0.2900+00	0.1270+00	-0.5310+00
0.3090+00	0.9550-01	0.1000+01	-0.4710+00	0.3690+00	0.1480+00	-0.7710+00
-0.5290-01	0.6710+00	-0.4710+00	0.1000+01	-0.7170-02	0.3480-03	0.6770-02
0.7140+00	0.2900+00	0.3690+00	-0.7170-02	0.1000+01	0.2210+00	-0.4800+00
0.4850+00	0.1270+00	0.1480+00	0.3480-03	0.2210+00	0.1000+01	-0.2260+00
-0.3840+00	-0.5310+00	-0.7710+00	0.6770-02	-0.4800+00	-0.2260+00	0.1000+01

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